Tests of Relativistic Gravity using Millisecond Pulsars

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ABSTRACT

General relativity asserts that: energy and momentum conservation laws are valid, preferred frames do not exist, and the strong equivalence principle is obeyed. In this paper recent progress in testing these important principles using millisecond pulsars is summarised.

1. Introduction

The fundamental physics and principles that can be observed and tested by the exceptional precision of pulsar timing includes (Bell 1997):

- Relativistic precession
- Shapiro delay
- Einstein delay
- Gravitational waves
- Variation in G
- Chandrasekhar mass
- Spin-orbit coupling
- Ultra low frequency gravitational waves
- Strong equivalence principle
- Lorentz Invariance
- Conservation laws

At this meeting Esposito-Farèse gave an update on the first 4 items and summary of the parametrised post-Newtonian formalism (PPN). Will (1993) also discusses the PPN formalism in detail and gives limits on many of the ten PPN parameters. Taylor et al. (1992) discuss many other relativistic effects which could in principle be measured with sufficient precision. Limits on the PPN parameters α_1, α_2, α_3 and ξ will be discussed here in relation to the last two items. Tests of the strong equivalence principle (SEP) giving limits on Δ will also be discussed due to the similar nature of the tests. These tests are null tests and it is the 90% confidence level limits which are quoted.

In placing such limits, one wishes to know the extent to which strong field effects are contributing. Measurement of a given PPN parameter $\hat{\alpha}$ contains both a weak field contribution α and a strong field contribution $\alpha^{'}$ (Damour & Esposito-Farèse 1992)

$$\hat{\alpha} = \alpha + \alpha^{'}(c_1 + c_2 + \cdots) + \cdots \qquad (1)$$

Here c_1, c_2 represent the compactness (E_{grav}/mc^2) of the bodies involved. For the sun, $c_i \sim 10^{-6}$, for a neutron star $c_i \sim 0.2$ and for a black hole $c_i \sim 0.5$. Therefore, strong field effects are poorly constrained by solar system experiments, while pulsars provide comparable sensitivity and ease of study when compared to black holes.

The cosmic microwave background (CMB) has been chosen as the absolute frame in most studies. While some recent results (Lauer & Postman 1994) have questioned this, it is the magnitude, not the direction of the absolute velocity **w** that is most relevant; this is similar for both the CMB and Lauer & Postman data.

One might ask whether the similar nature (i.e. upper limits from low eccentricity orbits) of the tests discussed below (which constrain Δ, α_1 and α_3) makes them degenerate. This is not the case; there are sufficient degrees of freedom and different figures of merit for each test so that different pulsars are being used for each test.

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2. Lorentz Invariance, $|\alpha_2| < 2.4 \times 10^{-7}$

If a gravitational interaction is not Lorentz invariant (PPN $\alpha_2 \neq 0$, due to some long-range tensor field), an oblate spinning body will feel a torque (Nordtvedt 1987):

$$\tau \propto \alpha_2 \mathbf{w} \times \mathbf{\Omega}, \tag{2}$$

where $\mathbf{\Omega}$ is the angular velocity. This torque would cause the spin axis to precess about $\mathbf{w}$. Since the spin-orbit coupling between the sun and planets is weak, the close alignment ($\sim 6^\circ$) of the spin and orbital angular momenta means that the above torque is weak. Quantitatively, the limit is $|\alpha_2| < 2.4 \times 10^{-7}$, showing that the gravitational interaction is Lorentz invariant to high precision (Nordtvedt 1987).

There are two important assumptions made here: primordial alignment of the spin and orbital angular momenta and that the sun has not made many rotations and by chance is closely aligned at the present epoch. Pulsars play a role in confirming that the second assumption is valid since if the torque was sufficiently large to cause the sun to make many rotations it would also be large enough to cause the fastest pulsars to precess out of view (Nordtvedt 1987), assuming they do not have fan beams.

Nordtvedt (1987) also considered preferred location effects as distinct from the above preferred frame effects. The resulting Lagrangian for a three-body interaction contains the PPN parameter ξ. Using the Galactic center as a distant third body yields $\tau \propto \xi \mathbf{w} \times \mathbf{\Omega}$, giving a limit on ξ similar to the limit on α_2 by the same arguments.

3. Polarised orbits and Relativistic Precession

The tests discussed in Sections 4, 5 and 6.2 search for the presence of eccentricities induced in pulsar orbits. These are gravitational analogues of the Stark effect, with the orbits being polarised in particular directions. However there is a non-zero probability that the relativistic precession of the orbit may cause cancellation with the intrinsic eccentricity of the system.

The problem of the possible cancellation was first considered by Damour and Schäfer (1991). They noted that, if the binary pulsar system is old compared to the time scale for precession, so that many rotations had been completed, then a statistical treatment of the probability of cancellation was sufficient since the goal was an upper limit rather than a measurement. A more precise statistical treatment was derived by Wex (1996) who also demonstrated the power of using multiple systems to improve the limits.

4. Strong Equivalence Principle, $|\mathbf{\Delta}| < 0.004$

The SEP requires the universality of the free fall of self-gravitating objects, i.e. in the same gravitational potential, two bodies should feel the same acceleration regardless of their mass, composition and density. Nordtvedt (1968) showed that if SEP did not hold for the Earth-Moon-Sun system, the Moon's orbit would be eccentric and polarised with the semi-major axis pointing towards the Sun. So began the now famous lunar-laser-ranging experiments which searched for this polarisation using the Apollo 11 reflector and measurement uncertainties of ~ 1 ns in the time of flight.

Damour and Schäfer (1991) pointed out the need for such a test in a strong field regime and showed that it is possible using binary pulsars. They suggested that the Earth-Moon-Sun system be replaced with a pulsar-companion-Galaxy system. If the companion is a white dwarf, the composition, density and self gravity is very different to the pulsar giving sensitivity to strong field effects. Damour and Schäfer (1991) showed that the figure of merit for choosing the best test systems is $f_\Delta = P_b^2/e$ and used PSR B1953+29 to obtain the limit $|\Delta| = |1 - M_I/M_G| < 0.01$. Arzoumanian (1995) suggested that PSR B1800–27 could be used to improve this limit to $|\Delta| < 0.004$, however it is not clear that this system is sufficiently old (Wex 1996). If several pulsars are used simultaneously the multiplication of small probabilities leads to the very rigorous bound of $|\Delta| < 0.004$ (Wex 1996).

5. Lorentz Invariance, $|\hat{\alpha}_1| < 1.7 \times 10^{-4}$

If preferred reference frames exist and $\alpha_1 \neq 0$, then there is a constant forcing term in the time evolution of the eccentricity vector of a binary system. For a very low eccentricity orbit, this tends to “polarize” the orbit, aligning the eccentricity vector with the projection onto the orbital plane of the absolute velocity of the system. Hence, the orbital parameters of very low eccentricity binary pulsars such as PSR B1855+09 may be used to set an upper bound of $|\hat{\alpha}_1| < 5 \times 10^{-4}$ (Damour & Esposito-Farèse 1992). This compares with limits from solar system data of $\alpha_1 = 2.1 \pm 1.9 \times 10^{-4}$ (Hellings 1984).

The most circular orbit known ($e \sim 10^{-6}$), that of PSR J2317+1439 (Camilo, Nice, & Taylor 1996) has a figure of merit $f_{\alpha_1} = P_b^{1/3}/e$ 10 times better than PSR B1855+09. However, the more unfortunate orientation with respect to the CMB and poorly constrained radial velocity means that only a factor of 3 improvement was possible, giving a limit of $|\hat{\alpha}_1| < 1.7 \times 10^{-4}$ (Bell, Camilo, & Damour 1996).

6. Conservation Laws and Lorentz Invariance, $|\alpha_3| < 2.2 \times 10^{-20}$

As shown by Nordtvedt and Will (1972) , a non-zero α_3 induces a contribution to the perihelion precession of the planets in the solar system. The two planets with the best measurements of periastron advance were Earth and Mercury. By combining the observations for two planets it is possible to eliminate the terms involving other parameters, obtaining $|49\alpha_1 - \alpha_2 - 6.3 \times 10^5 \alpha_3 - 2.2\xi| < 0.1$ (Will 1993). Using the limits on α_1, α_2, ξ a limit of $|\alpha_3| < 2 \times 10^{-7}$ was thus obtained.

6.1. Single Pulsars

A tighter limit on α_3 has been obtained by considering the effect of the acceleration

$$\mathbf{a}_{self} \propto \alpha_3 \mathbf{w} \times \mathbf{\Omega} \tag{3}$$

on the observed pulse periods of isolated pulsars. The observed pulse period $P \simeq P_0(1+v_r/c)$, contains a contribution from the Doppler effect due to the radial velocity v_r. Similarly any radial acceleration a_r contributes to the observed period derivative $\dot{P} \simeq \dot{P}_0 + P a_r/c$.

Self accelerations are directed perpendicular to both $\mathbf{w}$ and $\mathbf{\Omega}$. If self accelerations were contributing strongly to the observed period derivatives of pulsars, roughly equal numbers of positive and negative observed period derivatives would be expected, since the spin axes and therefore the self accelerations are randomly oriented.

The observed distribution of normal pulsars (excluding those pulsars in globular clusters) contains only positive period derivatives, allowing a limit of $|\alpha_3| < 2\times10^{-10}$ to be placed (Will 1993). Using millisecond pulsars, Bell (1996) obtained a limit of $|\alpha_3| < 5 \times 10^{-16}$.

6.2. Binary Pulsars

For a binary pulsar with a white dwarf companion, we again have two bodies with very different self gravities and sensitivities to strong field effects. If $\alpha_3 \neq 0$, the induced self-acceleration of the white dwarf would be negligible compared to that of the pulsar. Hence, we now have a rocket in a binary system. Since $\mathbf{a}_{self}$ is perpendicular to $\mathbf{\Omega}$ and since the spin and orbital angular momenta are aligned for recycled systems, the $\mathbf{a}_{self}$ is in the plane of the orbit. The resulting effect is a polarised orbit similar to those predicted by SEP violations (Section 4) and Lorentz invariance violations (Section 5) (Bell & Damour 1996).

The figure of merit for choosing the best test systems is $f_{\alpha_3} = P_b^2/eP$. Selecting appropriate systems and applying the statistical treatment of relativistic precession (Section 3) gives a limit of $|\alpha_3| < 2.2 \times 10^{-20}$ (Bell & Damour 1996). The only other ultra-high-precision null experiments (giving limits of order 10^{-20} on a dimensionless theoretical parameter) of which we are aware, are the recent Hughes-Drever-type tests (Prestage et al. 1985, Lamoreaux et al. 1986, Chupp et al. 1989), shown in Figure 14.2 of Will (1993). It is remarkable that tests involving binary pulsars can rank, together with modern laser-cooled trapped atom experiments, among the most precise null experiments of physics.

7. Prospects for Further Improvements

The figures of merit indicate how strongly these tests depend on the orbital periods and eccentricities of binary pulsars. The dotted lines in Figure 1 indicate the relative slopes of $f_\Delta \propto P_b^2/e$ and $f_{\alpha_1} \propto P_b^{1/3}/e$. There are also strongly evolutionary links expected between P_b and e (Phinney 1992) as shown by the solid line. Comparison of the slope of this curve, with the figure of merit dependence on P_b and e indicates that scope for improvements of the SEP test is small unless more longer orbital period systems can be found. A similar conclusion for the α_3 limit can be drawn but the additional dependence on P, ($f_{\alpha_3} \propto P_b^2/eP$) makes it less clear.

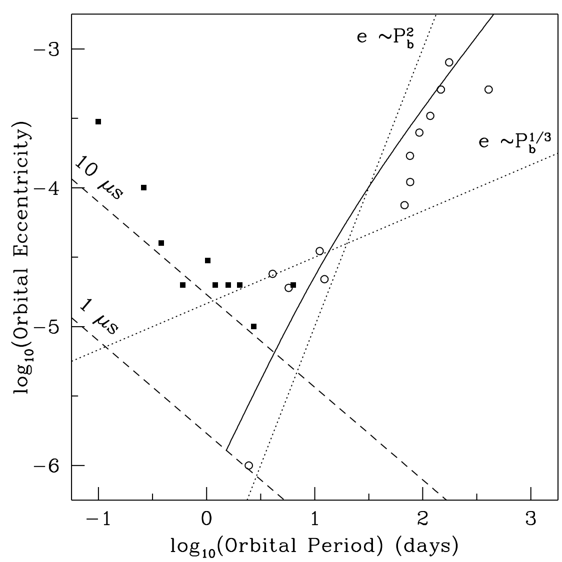

Fig. 1.— Orbital eccentricities of radio pulsars. Circles — low-mass binary pulsars. Dashed lines – approximate limits obtainable on eccentricities for rms timing residuals of $10\,\mu$s and $1\,\mu$s.

The flatter dependence on P_b of f_{α_1} means that short orbital period systems are preferable. However, only upper limits are presently available for many of these (Figure 1). If these upper

limits could be reduced substantially, it should be possible to usefully improve the limit on α_1, especially if several systems are used.

The limits on α_2 and ξ may be improved slightly, by careful consideration of pulse profile changes of the fastest pulsars to obtain limits on the precession. However there is another more promising approach. If the orientation of the spin and orbital angular momenta could be determined for binary millisecond pulsars it would be possible to improve the limits on α_2 and ξ by several orders of magnitude. The inclination of the PSR J1012+5307 orbit has been determined from optical observations (van Kerkwijk, Bergeron, & Kulkarni 1996). It may be possible to obtain the orientation of the pulsar spin axis from polarisation observations. With only one system, the ambiguity of many rotations would remain remain, but with several such binary systems, statistical arguments similar to those used for Δ and α_3 could provide very strict limits.

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REFERENCES

Arzoumanian, Z. 1995. PhD thesis, Princeton University

Bell, J. F. 1996, ApJ, 462, 287

Bell, J. F. 1997, in Satellite and Ground Based Studies of Radio Pulsars, Proceedings of Section E1.5 of the 31st Scientific Assembly of COSPAR ed. P. Caraveo, Elsevier Science, In Press

Bell, J. F., Camilo, F., & Damour, T. 1996, ApJ, 464, 857

Bell, J. F. & Damour, T. 1996, Classical Quantum Gravity, 13 3121

Camilo, F., Nice, D. J., & Taylor, J. H. 1996, ApJ, 461, 812

Chupp, T.E. et al. 1989, PRL, 63, 1541

Damour, T. & Esposito-Farèse, G. 1992, PRD, 46, 4128

Damour, T. & Schäfer, G. 1991, PRL, 66, 2549

Hellings, R. W. 1984, in General Relativity and Gravitation, Proceedings of the Tenth International Conference, ed. B. Bertatti, F. de Felice, & A. Pascolini, Reidel, Dordrecht, 365

Lamoreaux, S. K. et al. 1986, PRL, 57, 3125

Lauer, T. R. & Postman, M. 1994, ApJ, 425, 418

Nordtvedt, K. 1968a, Phys. Rev., 169, 1014

Nordtvedt, K. 1968b, Phys. Rev., 170, 1186

Nordtvedt, K. 1987, ApJ, 320, 871

Nordtvedt, K. & Will, C. M. 1972, ApJ, 177, 775

Phinney, E. S. 1992, PTRSA, 341, 39

Prestage, J. D. et al. 1985, PRL, 54, 2387

Taylor, J. H., Wolszczan, A., Damour, T., & Weisberg, J. M. 1992, Nature, 355, 132

van Kerkwijk, M. H., Bergeron, P., & Kulkarni, S. R. 1996, ApJ, 467, L89

Wex, N. 1997, A&A. In Press

Will, C. M. 1993, Theory and Experiment in Gravitational Physics, (Cambridge University Press)